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RELAXATION IN A LOW-PERMEABILITY POROUS MATERIAL

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Relaxation in low-permeability porous materials has been examined for nonstationary infiltration.

Darcy's law applies [1] to steady-state motion through an isotropic porous material under a low pressure gradient, which relates the flow speed **u** to the pressure p and the gravitational potential φ:

$$\mathbf{u} = -\frac{k}{\mu}\nabla G, \quad G = p + \rho\varphi. \tag{1}$$

(1) must be modified on a relaxation model for nonstationary infiltration [2, 3]. If **u** and G are variable in time, (1) is replaced for a homogeneous isotropic porous medium by

$$\mathbf{u}(t) = -\frac{k}{\mu}\int_{-\infty}^{+\infty} K(t-t_1)\nabla G(t_1)\,dt_1, \tag{2}$$

in which the kernel K(t) is independent of the spatial coordinates and satisfies the conditions: K = K(t) is a function of time with dimension 1/time, possibly generalized, $\int_{-\infty}^{+\infty} K(t)dt = 1$, and the support for K = K(t) by virtue of causality lies on the [0, +∞) semiaxis.

If one performs a Fourier transformation with respect to t for the functions in (2), one gets

$$\mathbf{u}(\omega) = -\frac{k}{\mu}\tilde{K}(\omega)\nabla G(\omega), \tag{3}$$

in which the complex function $\tilde{K}(\omega)$ is such that

$$\tilde{K}(0) = 1, \quad \overline{\tilde{K}(\omega)} = \tilde{K}(-\omega). \tag{4}$$

For small ω, the simplest approximation for $\tilde{K}(\omega)$ compatible with (4) is the linear one:

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$$\tilde{K}(\omega) = 1 + i\tau\omega, \tag{5}$$

in which τ is a real function with the dimensions of time. The inverse Fourier transformation from (3) on the basis of (5) gives the following particular form for (2):

$$\mathbf{u} = -\frac{k}{\mu}\left(1 + \tau\frac{\partial}{\partial t}\right)\nabla G. \tag{6}$$

Consider a horizontal cylindrical specimen having length L, into which liquid begins to be pumped from one end with a mass flow rate per unit area of q. At the other end, a constant pressure p_0 is maintained. We use (6) to examine the pressure at the input.

The spatial coordinate x is reckoned along the specimen and varies over the segment [0, L]. The continuity $\partial m\rho/\partial t + \partial\rho u/\partial x = 0$ and (6) for a weakly compressible liquid give

$$\frac{\partial p}{\partial t} = \frac{kE}{m\mu}\left(1 + \tau\frac{\partial}{\partial t}\right)\frac{\partial^2 p}{\partial x^2}. \tag{7}$$

The boundary and initial conditions are

$$-\rho\left(1 + \tau\frac{\partial}{\partial t}\right)\frac{k}{\mu}\frac{\partial p}{\partial x}\Big|_{x=0} = q, \tag{8}$$

$$p|_{x=L} = p_0, \tag{9}$$

$$p|_{t=0} = p_0. \tag{10}$$

We put $\varkappa = kE/m\mu$, $\alpha = q\mu/\rho k$, and then (8) and (10) give an expression for $\frac{dp}{dx}\Big|_{x=0}$ as a time function:

$$\frac{\partial p}{\partial x}\Big|_{x=0} = -\alpha\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]. \tag{11}$$

To solve (7) with (9), (10), and (11), we define an auxiliary function v = v(t, x) from

$$p(t, x) = p_0 - \alpha\left[1 - \exp\left(-\frac{t}{\tau}\right)\right](x - L) + v(t, x). \tag{12}$$

We substitute (12) into (7), (9), (10), and (11) to get the following equation for v = v(t, x):

$$\frac{\partial v}{\partial t} = \varkappa\left(1 + \tau\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial x^2} + \frac{\alpha}{\tau}(x - L)\exp\left(-\frac{t}{\tau}\right), \tag{13}$$

$$v|_{t=0} = 0, \quad \frac{\partial v}{\partial x}\Big|_{x=0} = 0, \quad v|_{x=L} = 0.$$

Let $A = \partial^2/\partial x^2$ be a self-conjugate operator acting in the space X of functions integrable in square in the segment [0, L] and satisfying $\frac{\partial v}{\partial x}|_{x=0} = 0$, $v|_{x=L} = 0$. Then (13) gives a linear inhomogeneous Cauchy problem in the linear space X:

$$\frac{\partial v}{\partial t} = Bv + \frac{\alpha}{\tau}(1 - \varkappa\tau A)^{-1}(x - L)\exp\left(-\frac{t}{\tau}\right), \tag{14}$$

$$v|_{t=0} = 0, \quad B = \varkappa A(1 - \varkappa\tau A)^{-1}.$$

A simple solution applies for (14):

$$v = \alpha\left[\exp(tB) - \exp\left(-\frac{t}{\tau}\right)\right](x - L),$$

and then (12) gives p as:

$$p = p_0 - \alpha(x - L) + \alpha\exp(tB)(x - L). \tag{15}$$

(15) shows that the pressure at x = 0 is

$$p = p_0 + \alpha L + \alpha\sum_{n=0}^{+\infty} C_n f_n(0)\exp(tb_n),$$

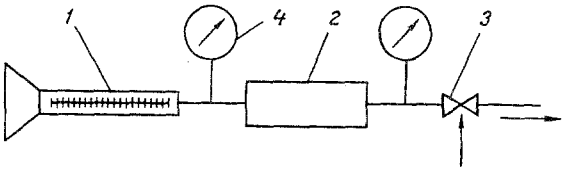

Fig. 1. Apparatus.

TABLE 1. Nonstationary Liquid Infiltration

Liquid	Q, 10^{-8} m^3/sec	$\bar{p}$, MPa	k, 10^{-15} m^2	τ', sec	τ_0, sec	τ, sec
Acetone	6,67	19,90	13,6	44	666	622
Acetone	3,40	16,25	13,5	44	614	570
Acetone	0,22	12,65	13,6	44	604	560
Toluene	0,083	12,58	13,0	91	700	609
Toluene	1,67	15,85	13,3	89	760	671
Toluene	3,33	19,40	13,1	90	800	710
Benzene	0,083	12,65	11,1	104	1010	906
Benzene	2,05	18,05	11,1	104	1030	926
Benzene	2,76	20,05	11,1	104	1090	986

in which $f_n = f_n(x)$, $n = 0, 1, \ldots$, are normalized eigenfunctions for A corresponding to certain eigenvalues λ_n, while $b_n = \varkappa\lambda_n/(1-\varkappa\tau\lambda_n)$ are the eigenvalues for the operator B, and the coefficients C_n arise in the expansion $(x-L) = \sum_{n=0}^{+\infty} C_n f_n(x)$. The spectrum λ_n, $n = 0, 1, \ldots$, for A is found in a standard fashion [4] and is $\lambda_n = -\left(\frac{\pi}{2} + \pi n\right)^2 L^{-2}$, $n = 0, 1, \ldots$, so there is a relaxation spectrum for the pressure at x = 0 with characteristic times

$$\tau_n = -1/b_n = \tau + \frac{4L^2}{\varkappa\pi^2(1+2n)^2}, \quad n = 0, 1, \ldots$$

The principal relaxation time is

$$\tau_0 = \tau + \tau', \tag{16}$$

in which $\tau' = 4L^2/\varkappa\pi^2$, and contains the relaxation time in a particle τ together with a component τ' that can be calculated if k and m are known from independent experiments. Then one determines τ_0 by experiment to derive the characteristic time for the internal relaxation.

This method has been used in a series of experiments with the Fig. 1 apparatus. The cylinder 2 consists of packed crushed quartz sand, L = 5 m, diameter $d = 2.6\cdot10^{-2}$ m, porosity m = 0.222. At one end, the measurement press 1 is connected, whose piston is displaced at a set speed. The fine-regulation valve 3 at the exit maintains a constant pressure by gradually draining the liquid. The liquid is forced through at a consant flow rate Q, and the control valve maintains a constant p_0, while the varying pressure p(t) in the inlet line is recorded by the gauge 4. When the pressure ceases to rise, the stationary state has been attained, with the permeability then measured in a standard fashion there. One determines τ_0 by processing p(t).

We used acetone, toluene, and benzene with various mean pressures $\bar{p}$. Table 1 gives the conditions. The τ were calculated from (16). We took for acetone $E = 1.22\cdot10^9$ Pa, $\mu = 3.25\times10^{-4}$ Pa·sec, for toluene $E = 1.11\cdot10^9$ Pa, $\mu = 5.84\cdot10^{-4}$ Pa·sec, and for benzene $E = 1.27\cdot10^9$ Pa, $\mu = 6.52\cdot10^{-4}$ Pa·sec.

The measurements always gave $\tau_0 \gg \tau'$, so the relaxation cannot be explained by the (1) model.

Charnyi [5] pointed out that there can be increased pressure permeability in nonstationary flow because of residual gas. Also, the effect may be due to small parasitic volumes in the apparatus. Therefore, provision was made for outgassing the liquid and evacuating the entire system before liquid saturation; gas bubble effects were thereby excluded. In the interpretation, we also considered mechanical relaxation in the equipment due to the changing pressure: in the seals, elastic components, etc. to estimate the latter, we made tests which showed that relaxation there was unimportant in the system without the porous medium.

Darcy's law does not apply to nonstationary infiltration into a low-permeability porous material, and (6) must be used. The relaxation time is a characteristic of the porous medium and fluid and is also dependent on the pressure. The relaxation times usually considered in the theory [2, 3] are $\leq 10^3$ sec, and we have demonstrated for the first time in a laboratory experiment that there is relaxation times of 10^2-10^3 sec.

NOTATION

ρ and ρ_0, densities; p and p_0, pressures; E, bulk modulus of elasticity for the liquid; u, infiltration speed; k, permeability; m, porosity; μ, viscosity; $\varkappa$, pressure conductivity; φ, gravitational potential; $K(t)$, relaxation kernel; t, time; ω, complex variable, argument in the Fourier transformation; $\tilde{K} = \tilde{K}(\omega)$, Fourier transformation for the relaxation kernel; f_n, normalized eigenfunctions for operator A; λ_n, spectrum for A; A, self-conjugate operator in space X; X, space of functions integrable in squares; B, operator; b_n, eigenvalues of B; C_n, series expansion coefficients; q, mass flow rate per unit area of porous medium; Q, volume flow rate; v, auxiliary function defined by (12); τ, τ_n, τ', relaxation times; L, specimen length; d, diameter.

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